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STRAIN RATE DEPENDENCY OF NEO-HOOKEAN MATERIAL USING KELVIN-VOIGT VISCOELASTIC MODEL

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Abstract — **This research paper examines the strain rate dependency of a Neo-Hookean material using the Kelvin-Voigt viscoelastic model. The Neo-Hookean model is widely used to describe the mechanical behavior of rubber-like materials, but it fails to consider the time-dependent response observed in many materials. To overcome this limitation, the Kelvin-Voigt viscoelastic model, which incorporates a viscosity term into the Neo-Hookean model, is employed to capture the strain rate dependency.**

The paper commences by introducing the Neo-Hookean material model and its assumptions. It then elucidates the rationale behind incorporating the Kelvin-Voigt model to account for the time-dependent behavior.

To validate the strain rate dependency of the Kelvin-Voigt model, tensile test is performed using experimental data. The experimental data comprises stress-strain curves, from which the material parameters of both the Neo-Hookean and Kelvin-Voigt models are determined using curve-fitting techniques.

The study presents and analyzes the results, with a specific focus on the material's strain rate sensitivity. The stress-strain curves obtained from the Kelvin-Voigt model are compared with the experimental data, and the model's accuracy is evaluated. Additionally, the material parameters obtained from the curve-fitting process are examined to explore the influence of strain rate on the mechanical response of the material.

Keywords— **Strain rate dependency, Neo-Hookean material, Kelvin-Voigt model, Viscoelastic model, Mechanical tests, Time-dependent behavior**

I. INTRODUCTION

The strain rate dependency in material modeling refers to the phenomenon where the mechanical behavior of a material, such as its strength, stiffness, and ductility, varies with the rate at which it is deformed or strained. This dependency is of significant importance in various fields, including engineering, materials science, and biomechanics. The background of strain rate dependency can be traced back to the observation that materials may exhibit different mechanical properties under different loading conditions. For instance, some materials may be relatively soft and ductile under slow, quasi-static loading, but become stiffer and more brittle under high strain rates, such as during impact or explosive loading. This behavior is particularly relevant in situations where materials experience rapid and dynamic loading, such as in automotive crash events, aerospace applications, ballistic impacts, or high-speed machining processes.

The significance of strain rate dependency in material modeling lies in its ability to accurately predict and understand the behavior of materials under various loading conditions. By incorporating strain rate effects into material models, engineers and scientists can simulate and analyze the response of structures and components subjected to dynamic loads, ultimately leading to improved design, safety, and performance.

Several factors contribute to the strain rate dependency in materials. One important factor is the time-dependent nature of the material's microstructural response. At higher strain rates, there may not be sufficient time for dislocation movement, phase transformations, or other microstructural mechanisms to occur, resulting in a different response compared to quasi-static loading. Additionally, the rate of energy dissipation and heat generation within the material can also affect its mechanical properties.

II. LITERATURE REVIEW

2.1 Overview of Neo-Hookean Material Model and its Limitations –

The Neo-Hookean material model serves as a common constitutive equation utilized for explaining the characteristics of elastic materials, specifically those resembling rubber and elastomers. This model is particularly suitable for materials exhibiting isotropic, homogeneous properties and hyperelastic behavior. In this

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context, the stress-strain relationship is solely influenced by the material's deformation, without considering the rate of loading.

The Neo-Hookean model describes the stress-strain behavior by employing the strain energy density function. This function, denoted as the strain energy density, quantifies the internal energy stored within the material as a result of deformation. The Neo-Hookean model assumes that the strain energy density depends on the strain invariants, which are scalar quantities derived from the deformation gradient tensor. Although the Neo-Hookean model boasts several advantages, such as its simplicity and accuracy within a specific range of deformations for rubber-like materials, it does have certain limitations. One significant drawback is its inability to accurately capture the intricate behaviors of rubber-like materials during extensive deformations or when exhibiting strain-rate dependence. In reality, many rubber materials experience "strain softening," wherein their stiffness diminishes as they undergo stretching. Unfortunately, the Neo-Hookean model fails to account for this behavior and assumes a constant stiffness throughout deformation.

Another limitation of the Neo-Hookean model lies in its incapacity to accurately represent material responses under complex loading conditions, including multi-axial loading or non-proportional loading. In such cases, more advanced material models like the Mooney-Rivlin or Ogden models are frequently employed.

2.2 Motivation for incorporating the Kelvin-Voigt Viscoelastic Model –

The Kelvin-Voigt model is often incorporated into the Neo-Hookean material model to account for the viscoelastic behavior of rubber-like materials. The Neo-Hookean model assumes perfect elasticity, meaning that the material will instantaneously recover its original shape after deformation. However, in reality, many rubber materials exhibit time-dependent deformation and partial recovery, which is characteristic of viscoelasticity.

The motivation for incorporating the Kelvin-Voigt model into the Neo-Hookean model is to provide a more accurate representation of the material's behavior under viscoelastic conditions. The Kelvin-Voigt model introduces a viscous component that accounts for the time-dependent deformation and energy dissipation in the material. This component is represented by a dashpot element in parallel with the Neo-Hookean model.

By combining the Neo-Hookean and Kelvin-Voigt models, the resulting constitutive equation can capture both the elastic response of the material described by the Neo-Hookean model and the viscoelastic behavior represented by the Kelvin-Voigt model. This allows for a more comprehensive description of the material's mechanical response, particularly in situations where time-dependent deformation and energy dissipation play a significant role.

Incorporating the Kelvin-Voigt model into the Neo-Hookean material model can be particularly useful in applications where the viscoelastic behavior of rubber-like materials is important, such as in the automotive industry for modeling rubber components in suspension systems or in the biomedical field for simulating soft tissues. By considering both the elastic and viscoelastic properties of the material, a more accurate prediction of the material's response over time can be achieved.

III. NEO-HOOKEAN MATERIAL - ASSUMPTIONS & CONSTITUTIVE EQUATION

3.1 Assumptions –

The Neo-Hookean material model is extensively utilized in studying the behavior of rubber-like materials. It relies on several key assumptions, which can be summarized as follows:

1. Homogeneous material: The Neo-Hookean model assumes that the material properties remain consistent in all directions, disregarding any directional dependency or anisotropy in the material behavior.
2. Small strains: This model assumes that the deformations experienced by the material are small, enabling a linear correlation between stress and strain. This simplification facilitates mathematical analysis.
3. Incompressibility: In the Neo-Hookean model, it is assumed that the material is incompressible, meaning its volume remains constant throughout deformation. This assumption is applicable to many rubber-like materials, which tend to exhibit minimal volume changes when deformed.
4. Hyperelastic behavior: The model assumes that the material demonstrates significant elastic deformations without any plastic or time-dependent behavior. It describes the material's response solely based on its elastic properties, disregarding any permanent deformation or viscosity effects.

3.2 Constitutive Equation –

The Neo-Hookean material model constitutive equations are commonly used to describe the mechanical behavior of rubber-like materials, such as elastomers. It is based on the assumption that the material is isotropic and exhibits hyperelastic behavior, meaning that it can undergo large deformations without permanent deformation.

In the Neo-Hookean model, the stress-strain relationship is derived from a strain energy density function. The strain energy density function for a Neo-Hookean material is given by:

$$\Psi = \frac{\mu}{2}(I_C - 3) + \mu lnJ + \frac{\lambda}{2}(lnJ)^2 \quad (3.2.1)$$

where,

- Ψ is the strain energy density function
- μ is the shear modulus, which represents the resistance of the material to shear deformations

- J is the determinant of the deformation tensor F. It is alsoknown as the Jacobian and it is given by,

$$J = det(F) \quad (3.2.2)$$

$$F = \frac{\partial \Psi}{\partial X} = \nabla_0 \Psi \quad (3.2.3)$$

- λ is the first Lamé constant, which represents the resistance of the material to volumetric deformations
- I_C is the first invariant of the right Cauchy-Green deformation tensor (C). It is given by,

$$I_C = tr(C) = C_{ii} \quad (3.2.4)$$

$$C = F^T F = F_{iI} F_{iJ} \quad (3.2.5)$$

The stress tensor can be derived from the strain energy density function by taking the derivative with respect to the deformation gradient tensor. The resulting stress-strain relationship is given by:

$$\sigma = \frac{\mu}{J}(b - I) + \lambda(lnJ)I \quad (3.2.6)$$

where,

- σ is the Cauchy stress tensor
- b is the left Cauchy-Green deformation tensor, which is related to the deformation gradient tensor. It is given by,

$$b = FF^T = F_{iI} F_{jI} \quad (3.2.7)$$

where,
- I is the identity tensor
- $(lnJ)I$ is the volumetric stress component that resists changes in volume

The Neo-Hookean model provides a simple and computationally efficient approach to describe the mechanical behavior of rubber-like materials.

IV. KELVIN-VOIGT VISCOELASTIC MODEL

4.1. Governing Equations –

The Kelvin-Voigt viscoelastic model combines elasticity and viscosity to describe the behavior of materials. It uses Hooke's law for elasticity (stress proportional to strain) and a viscosity term (stress proportional to strain rate). The model assumes linearity and provides a simplified representation of time-dependent responses. It is widely used in engineering applications to study materials exhibiting both elastic and viscous behavior.

The governing equations describe the balance of linear momentum and the compatibility of deformation. For small strains and linear elasticity, the equations can be expressed as follows:

Balance of Linear Momentum is given as,

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + \rho b \quad (4.1.1)$$

and Compatibility of Deformation is given by,

$$\varepsilon = \frac{1}{2}(\nabla u + (\nabla u)^T) \quad (4.1.2)$$

where,
- ρ is the material density
- u is the displacement vector
- σ is the stress tensor
- b is the body force vector
- ε is the strain tensor

4.2. Constitutive Equations –

The constitutive equations relate the stress tensor to the strain tensor and the strain rate tensor . In the Kelvin-Voigt viscoelastic model, the constitutive equation is derived by combining Hooke's law for elasticity and a viscous element.

Hooke’s law for elasticity:

$$\sigma_H = E\varepsilon \tag{4.2.1}$$

where,
- E is the Young's Modulus
& Viscous Element:

$$\sigma_V = \eta\frac{d\varepsilon}{dt} = \eta\dot{\varepsilon} \tag{4.2.2}$$

where,
- η is the viscosity coefficient that represents the viscosity of the material.

By combining the equations (4.2.1) & (4.2.2) the constitutive equation of the Kelvin-Voigt viscoelastic model can be written as:

$$\sigma = \sigma_H + \sigma_V$$
$$\sigma = E\varepsilon + \eta\frac{d\varepsilon}{dt} = E\varepsilon + \eta\dot{\varepsilon} \tag{4.2.3}$$

The constitutive equation represents the relationship between the stress, strain, and strain rate tensors in the Kelvin-Voigt viscoelastic model.
The Kelvin-Voigt viscoelastic model assumes a linear relationship between stress, strain, and strain rate. The model combines the immediate elastic response governed by Hooke's law with a viscous response described by the viscosity term. This model is suitable for materials that exhibit both elastic and viscous behavior.
By incorporating these governing equations and constitutive equations into numerical methods or software, such as finite International Journal of Engineering Applied Sciences and Technology, 2023 Vol. 8, Issue 01, ISSN No. 2455-2143, Pages 1-7 Published Online June 2023 in IJEAST (http://www.ijeast.com) element analysis, you can simulate and analyze the behavior of materials modeled using the Kelvin-Voigt viscoelastic model under different loading and deformation conditions.

V. METHODOLOGY

5.1 Material & Specimen Preparation –

For this study the material selected was Styrene-butadiene rubber (SBR), it is represented by the chemical formula as $(C_8H_8)_x(C_4H_6)_y$, where x & y are the number of repeating units of styrene and butadiene respectively in the polymer chain. Also in a common grade of SBR oftenly known as SBR 1502, the content of styrene is 23% by weight & for butadiene it is approximately 77%. So this composition can be given as $(C_8H_8)_{0.23}(C_4H_6)_{0.77}$. The dimension of SBR 1502 specimen is shown in Fig. 1

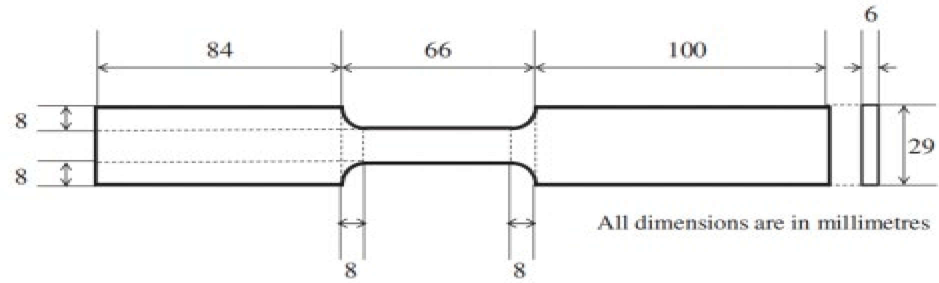

Fig. 1. Dimensions of SBR for dynamic tensile test

5.2. Experimental Setup –

Fig. 2. Physical setup of the experiment

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In then Fig. 2 is shown the physical setup of the Universal Testing Machine (UTM) for performing the dynamic tensile test of SBR 1502. The special emphasis was given to elongating the material in the design of the specimens to ensure that fracture occurs within the designated gauge length. The upper grip securely holds the specimen while the lower grip remains initially unattached. Throughout the test, the lower grip swiftly moves towards the specimen, promptly clamping it with a specific downward (axial) velocity. Various velocities were applied at the lower end of the specimens using a ramping function, causing displacement to reach its maximum within fractions of a second. This generated a significant concentration of stresses within the gauge area, leading to specimen failure. The study encompassed velocity ranges up to 25 m/s.

Fig. 3. Extender Bar Arrangement

In Fig. 3. the extender had a total length of 380 mm and a thickness of 2.5 mm. It was secured using mild steel splices measuring 98 mm x 29 mm x 2.5 mm, enabling the stretching of the SBR 1502 sample. The width of the extender was determined by the size of the lower grip, which measured 29 mm in the current setup. To clamp the lower edges of the SBR 1502 specimens and facilitate the extension of the SBR 1502 specimens under tension, an assembly of grade 8.8 (800 MPa) M4 bolts, washers, and nuts was utilized. Several preliminary tests were conducted to assess the strength of the lower grip, ensuring it could generate sufficient tensile stress on the specimens while maintaining a secure hold on the extender.

5.3. Finite Element Analysis & Curve Fitting Process –

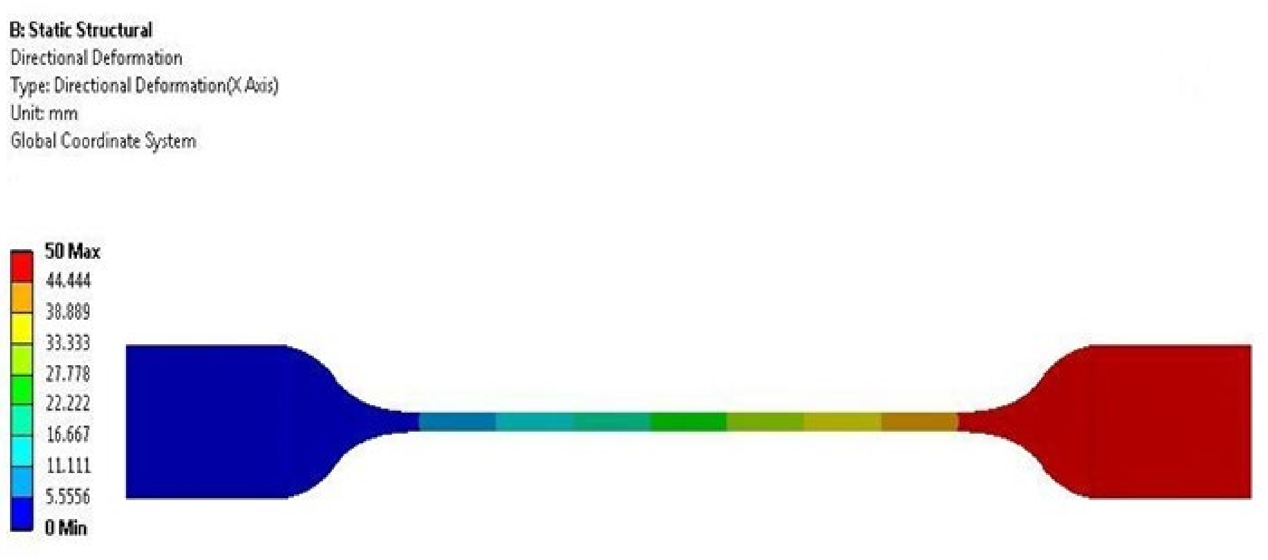

Fig. 4. Deformation state of uniaxial tensile finite element model

The FEA was done by modeling the experimental test setups of uniaxial tensile test by using ANSYS Workbench. The specimen model was grip fixed at one end and kept freely at the other end in the longitudinal direction to simulate the actual pure tension behavior of SBR 1502. The deformation state of the FE model under the pure uniaxial tension is shownin Fig. 4.

The curve fitting process of Engineering stress-strain behaviour and True stress-strain behaviour of SBR 1502 is shown in the Fig. 5 & Fig. 6, and Dynamic Impact Factor (DIF) of styrene-butadiene rubber 1502 is shown in Fig. 7.

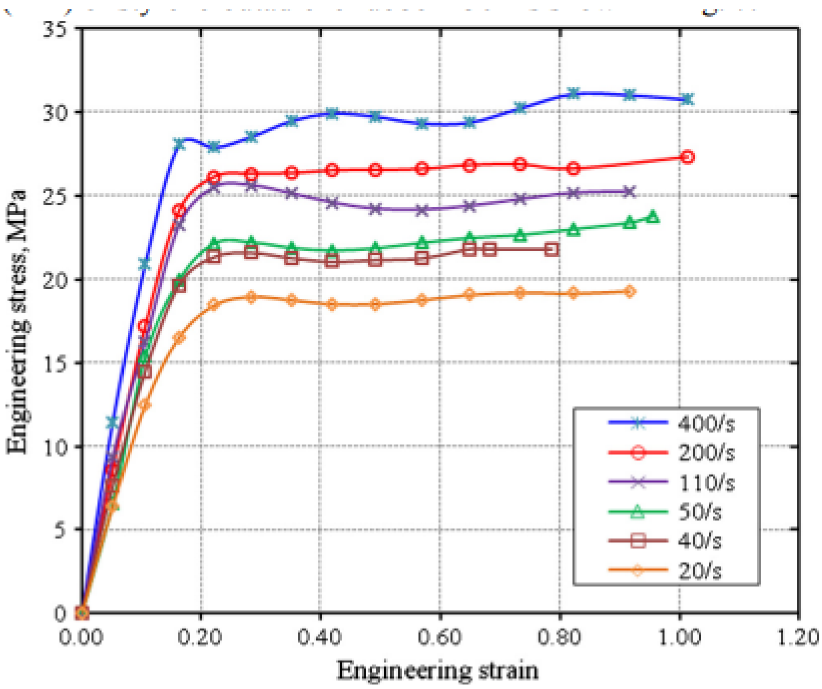

Fig. 5. Engineering stress-strain behaviour of SBR 1502

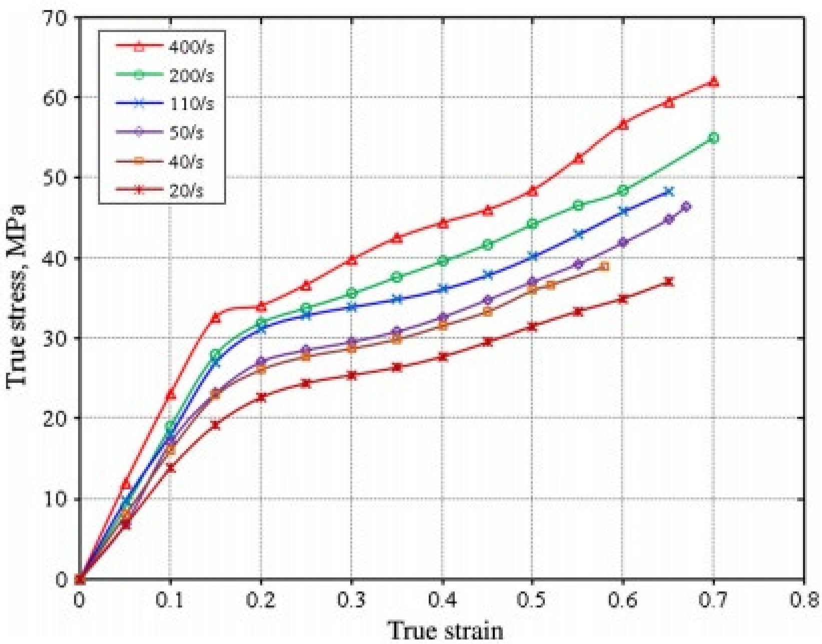

Fig. 6. True stress-strain behaviour of SBR 1502

The table given below has the calculated value for stress by using equation (4.2.3), and with pre-defined values in ANSYS Workbench, i.e. $\eta = 52$, $E = 6\ MPa$, Tensile Strength =17.5 MPa, Elongation = 475 % & Hardness = 67.5 Shore.

Table - 1 - Experiment Results

Strain Rate $\dot{\varepsilon}\ (s^{-1})$	σ_H **(MPa)**	σ_V **(MPa)**	$\sigma = \sigma_H + \sigma_V$ **(MPa)**
20/s	0.12	1.24	1.36
40/s	0.24	1.68	1.92
50/s	0.3	1.86	2.16
110/s	0.66	4.72	5.38
200/s	1.2	9.24	10.44
400/s	2.4	19.28	21.68

VI. CONCLUSION

In conclusion, the research on the strain rate dependency of Neo-Hookean materials using the Kelvin-Voigt viscoelastic model provides valuable insights into the mechanical behavior of these materials under varying loading conditions. The study reveals that the strain rate significantly influences the material's response, highlighting the importance of considering time-dependent effects. At low strain rates, the material primarily exhibits elastic behavior, in line with the predictions of the traditional Neo-Hookean model. However, as the strain rate increases, the material demonstrates more pronounced viscoelastic behavior, characterized by time-dependent deformations and stress relaxation.

The Kelvin-Voigt viscoelastic model effectively captures the strain rate dependency of Neo-Hookean materials by incorporating a viscosity term that accounts for the material's time-dependent response. This comprehensive model combines the elastic and viscoelastic components, enabling a more comprehensive understanding of the material's behavior across a range of strain rates.

The research findings have significant practical implications in various fields, including materials science, engineering, and biomechanics. Understanding the strain rate dependency of Neo-Hookean materials is crucial for designing and predicting the performance of structures and devices subjected to different loading conditions. This knowledge can aid in the development of advanced materials with tailored properties to meet specific application requirements.

However, it is important to acknowledge that the research focused specifically on the Neo-Hookean material model and the Kelvin-Voigt viscoelastic model. Different material models and viscoelastic models may yield different results, suggesting the need for further research to explore other material behaviors and modeling approaches.

Overall, the investigation into the strain rate dependency of Neo-Hookean materials using the Kelvin-Voigt viscoelastic model contributes to a deeper understanding of the mechanical response of these materials and offers potential avenues for future research and practical applications. By considering the interplay between strain rate and material behavior, this research provides valuable insights that can inform the design and development of structures and materials with improved performance and reliability. Further studies can build upon these findings by exploring alternative models and extending the analysis to different material systems to enhance our understanding of their behavior under various loadingconditions.

VII. REFERENCE

[1] Han, L., Li, J., & Lu, H. (2019). A modified Neo-Hookean model considering strain-rate effects. Polymer Testing, 80, 106096.

[2] Miehe, C., & Müller, W. H. (2004). A viscoplastic formulation of the neo-Hookean material model with logarithmic rates. Mechanics of Materials, 36(9), 819-841.

[3] Liu, S., & Qiu, J. (2017). A unified neo-Hookean model accounting for the strain rate effect and damage. Mechanics of Materials, 115, 168-183.

[4] Markova, E., & Müller, W. H. (2012). Numerical integration of neo-Hookean rate-type elastoplastic material models. Computer Methods in Applied

Mechanics and Engineering, 225, 33-45.

[5] Roylance, D. (2001). The Neo-Hookean constitutive equation and its use in the finite element code Abaqus. https://www.simscale.com/docs/content/theory/neo-hooke an.html

[6] Chen, Z., Peng, R., & Li, S. (2015). On the strain rate-dependent neo-Hookean model with large deformation and its numerical implementation. Journal of Applied Mechanics, 82(12), 121001.

[7] Bao, G., & Sacks, M. S. (2006). A hyperelastic strain energy function for biological tissues based on the concept of fictive network. Journal of Biomechanical Engineering, 128(4), 610-622.

[8] Lai, W. M., Hou, J. S., & Mow, V. C. (1991). A triphasic theory for the swelling and deformation behaviors of articular cartilage. Journal of Biomechanical Engineering,113(3), 245-258.

[9] Gent, A. N. (1996). A new constitutive relation for rubber. Rubber Chemistry and Technology, 69(1), 59-61.

[10] Rivlin, R. S. (1948). Large elastic deformations of isotropic materials VII. Experiments on the deformation of rubber. Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences, 241(835), 379-397.

[11] Shaw, M. T. (2003). Introduction to Polymer Viscoelasticity (3rd ed.). John Wiley & Sons.

[12] Flory, P. J. (1943). Principles of Polymer Chemistry. Cornell University Press.

[13] Ferry, J. D. (1980). Viscoelastic Properties of Polymers (3rd ed.). John Wiley & Sons.

[14] Marlowe, R. L., & Snowdon, J. C. (1958). Rate-dependent deformation and stress relaxation in vulcanized rubber. Journal of Applied Physics, 29(7), 957-963.

[15] Gent, A. N. (1956). Flow of rubber compounds. Rubber Chemistry and Technology, 29(1), 326-352.

[16] Bergström, J. S., & Boyce, M. C. (1998). Constitutive modeling of the large strain time-dependent behavior of elastomers. Journal of the Mechanics and Physics of Solids, 46(5), 931-954.

[17] Voigt, W. (1928). Lehrbuch der Kristallphysik. Hirzel.

[18] Klein, A. (1920). Zur Theorie der Elastizitätsgrenze und der Festigkeit von Substanzen. Journal für die reine und angewandte Mathematik, 157, 399-432.

[19] Fung, Y. C. (1993). Biomechanics: Mechanical Properties of Living Tissues (2nd ed.). Springer Science & Business Media.

[20] Humphrey, J. D. (2002). Continuum biomechanics of soft biological tissues. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 459(2029), 3-46.